# The Crystal Structure of $\left(\mathbf{P b}_{1-x}, \mathrm{Bi}_{x}\right) \mathrm{Bi}_{2} \mathrm{Cu}_{2} \mathrm{Cu}_{2-x} \mathrm{~S}_{5} \mathbf{I}_{2}(x=0.88)$ 

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The crystal structure of $\left(\mathrm{Pb}_{1-x}, \mathrm{Bi}_{x}\right) \mathrm{Bi}_{2} \mathrm{Cu}_{2} \mathrm{Cu}_{2-x} \mathrm{~S}_{5} \mathrm{I}_{2}(x=0.88)$ has been determined from three-dimensional intensity data. Four formula units of the compound are contained in the orthorhombic cell: $a=$ $13 \cdot 236$ (5), $b=23 \cdot 630$ (16), $c=4 \cdot 010$ (5) $\AA$ of the space group $B b m m$. The structure was derived from Patterson, Fourier and difference syntheses, and was refined by a least-squares method. The final $R$ for 725 independent reflexions is $7.4 \% .(\mathrm{Pb}, \mathrm{Bi})$ and Bi have three nearest sulphur atoms and form trigonal pyramids. One of the two independent copper atoms is trigonally coordinated with three sulphur atoms. Two sulphur and two iodine atoms are arranged tetrahedrally around the second copper atom. The occupancy of this copper atom is $56 \%$. The material has a layer structure, the layers being parallel to ( 010 ) and connected by weak $\mathrm{Bi}-\mathrm{I}$ bonds.

## Introduction

The compound $\mathrm{PbBi}_{2} \mathrm{Cu}_{4} \mathrm{~S}_{5} \mathrm{I}_{2}$ has been synthesized by transport reactions. As only a few structures of bismuth sulphiodides have been studied so far, the present investigation was undertaken to compare the coordination of iodine atoms around bismuth atoms in this material with iodine atoms in other bismuth sulphiodides.

## Experimental

The material was synthesized by chemical transport reaction from a mixture of appropriate proportion of $\mathrm{PbS}, \mathrm{Bi}_{2} \mathrm{~S}_{3}$ and $\mathrm{Cu}_{2} \mathrm{~S}$. The mixed suphides and 0.1 N HI solution were sealed in a silica glass tube and were kept at a temperature of $430^{\circ} \mathrm{C}$ with a gradient of about $15^{\circ} \mathrm{C}$ for 12 days. The crystals obtained were

Table 1. Crystal data

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(Pb
(x=0.88: determined from the structure)
Orthorhombic, space group Bbmm
a=13.236 \pm0.005,b=23.630\pm0.016,c=4.010\pm0.005 \AA
( }\lambda=1.5418\AA
Z=4
D calc}=6.570\mp@subsup{\textrm{g cm}}{}{-3
\mu=1322 cm
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needles of the length $0 \cdot 1-0.7 \mathrm{~mm}$, and of radius $0 \cdot 005$ 00.5 mm .

Semi-quantitative electron microprobe analysis of the crystals revealed that the content of $\mathrm{Pb}, \mathrm{Bi}$ and Cu varies for different crystals (Fig. 1) and that the chemical formula of these substances is expressed by $\mathrm{Pb}_{1-x} \mathrm{Bi}_{2+x} \mathrm{Cu}_{4-x} \mathrm{~S}_{5} \mathrm{I}_{2}(0 \leq x \leq 1)$.


Fig. 1. Variations of the parameter $x$ of $\mathrm{Pb}, \mathrm{Bi}$ and Cu for different crystals.

Table 2. Fractional coordinates, temperature factors in the expression $\exp \left[-\left(h^{2} \beta_{11}^{2}+k^{2} \beta_{22}^{2}+l^{2} \beta_{33}^{2}+2 h k \beta_{12}+2 h l \beta_{13}+2 k l \beta_{23}\right)\right]$ and occupancies

|  | Site | $x$ | $y$ | $z$ | $\beta_{11}$ | $\beta_{22}$ | $\beta_{33}$ | $\beta_{12}$ | $\beta_{13}$ | $\beta_{23}$ | $m$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ( $\mathrm{Pb}, \mathrm{Bi}$ ) | $8(f)$ | $0 \cdot 1058$ (2) | $0 \cdot 2407$ (2) | $\frac{1}{2}$ | 0.00184 (9) | $0 \cdot 00105$ (15) | 0.0143 (9) | $0 \cdot 00002$ (6) | 0 | 0 | $\frac{1}{2}$ |
| Bi | $8(f)$ | 0.0546 (1) | 0.0874 (1) | 0 | 0.00324 (6) | $0 \cdot 00051$ (2) | 0.0169 (5) | 0.00027 (3) | 0 | 0 | 1 |
| $\mathrm{Cu}(1)$ | $8(f)$ | 0.3671 (4) | $0 \cdot 1861$ (2) | 0 | $0 \cdot 00336$ (28) | $0 \cdot 00171$ (11) | 0.0167 (23) | $0 \cdot 00072$ (15) | 0 | 0 | 1 |
| $\mathrm{Cu}(2)$ | $8(f)$ | $0 \cdot 3159$ (6) | $0 \cdot 1098$ (4) | $\frac{1}{2}$ | $0 \cdot 00189$ (44) | $0 \cdot 00110$ (16) | 0.0685 (69) | 0.00022 (22) | 0 | 0 | $0 \cdot 560$ (13) |
| S(1) | 4(c) | $0 \cdot 2393$ (6) | $\frac{1}{4}$ | 0 | $0 \cdot 00057$ (40) | $0 \cdot 00170$ (21) | 0.0036 (39) | 0 | 0 | 0 | 1 |
| S(2) | $8(f)$ | $0 \cdot 1593$ (5) | $0 \cdot 1324$ (3) | $\frac{1}{2}$ | $0 \cdot 00216$ (34) | $0 \cdot 00081$ (11) | $0 \cdot 0126$ (29) | $0 \cdot 00003$ (16) | 0 | 0 | 1 |
| S(3) | $8(f)$ | $0 \cdot 4487$ (4) | $0 \cdot 1781$ (3) | $\frac{1}{2}$ | 0.00134 (32) | $0 \cdot 00068$ (11) | 0.0148 (29) | $0 \cdot 00007$ (15) | 0 | 0 | 1 |
| I | $8(f)$ | $0 \cdot 3629$ (1) | 0.0406 (1) | 0 | $0 \cdot 00264$ (11) | $0 \cdot 00073$ (3) | $0 \cdot 0130$ (9) | $0 \cdot 00010$ (5) | 0 | 0 | 1 |

The X-ray photographs of the crystals showed that the needles were always accompanied by small fragments of the same material, and that the $c$ axis of the fragments was almost parallel to that of the hosts $\left(\doteqdot 3^{\circ}\right)$. The crystal used for the determination of the lattice parameters and the intensities was a needle 0.4 mm long and 0.012 mm diameter. Some extra reflexions from a fragment were observed in oscillation photographs (\|c) of this crystal. Since these reflexions were

Table 3. Observed and calculated structure factors











 very weak, their effect on the structure determination was not taken into account. The cell dimensions were measured on a single-crystal diffractometer. The observed diffraction symmetry, mmmBb--, permits $B b m m$ (No. 63), Bbm2 (No. 40) and $B b 2_{1} m$ (No. 36) as possible space groups. The correct space group determined from the structure is Bbmm . The crystal data of the material are given in Table 1.

The intensities were measured from $h k 0$ to $h k 4$ on an automated Weissenberg diffractometer (STAD I-2 system from Stoe) using nickel-filtered copper radiation and were corrected for Lorentz and polarization effects. Absorption corrections for cylindrical shape were also applied. 74 of the 725 independent reflexions had $I<2 \sigma(I)$ and were designated unobservable reflexions.

## Determination and refinement of the structure

The reflexions from the layers with $l=2 n$ show approximately the same intensity distributions. The intensity distributions of the odd layers are similar to each other but not to that of the even layers. This characteristic of the intensity distribution suggested that almost all atoms should be on the levels with $z=0$ and $\frac{1}{2}$, and that $B b m 2$ could be omitted from the possible space groups. $B b 2_{1} m$ was also excluded by the interpretation of the Patterson projection, $P(u v)$, along the $c$ axis. At first, $\mathrm{PbBi}_{2} \mathrm{Cu}_{4} \mathrm{~S}_{5} \mathrm{I}_{2}$ was assumed to be the chemical formula of the specimen. The coordinates of $\mathrm{Pb}, \mathrm{Bi}$ and I were determined from the Patterson diagrams, $P(u v)$ and $P_{1}(u v)$ (generalized projection). Those of the other atoms were determined by successive Fourier and difference syntheses. The structure was refined by a least-squares method. The conventional $R$ index was $8.4 \%$ at this stage. During the process of refinement, two anomalies were noted: the temperature factor of $\mathrm{Cu}(2)\left(6.81 \AA^{2}\right)$; further, the $\beta_{22}$ component of the anisotropic temperature factor of Pb was extremely high $\left(\sqrt{u^{2}}=0.24 \AA\right)$. The occupancy of $\mathrm{Cu}(2)$ was then varied and the position for $\mathrm{Pb}[4(c) \mathrm{mm}]$ was divided into two positions $[8(f) m]$. Further refinement by a least-squares method reduced $R=\Sigma| | F \mathrm{Fobs}|-|F \mathrm{calc}|| /$ $\sum \mid F$ obs $\mid$ for all 725 reflexions to $7.4 \%$ and $R^{\prime}=$ $\left\{\sum w\left(\mid F \text { obs }|-|F \mathrm{calc\mid}|)^{2}\right\}^{1 / 2} /\left\{\sum w|F \mathrm{obs}|^{2}\right\}^{1 / 2}\right.$ to $6.5 \%$. The anomaly, formerly found in the difference map, disappeared thereafter. The occupancy of $\mathrm{Cu}(2)$, determined by the least-squares refinement, is $56 \%$ and the parameter $x$ of the chemical formula, calculated from this value, is 0.88 . At the final stage of the refinement, an atom form factor for bismuth was therefore used instead of that for lead.

A local version* of the full-matrix least-squares program ORFLS (Busing, Martin \& Levy, 1962) was used for structure-factor calculations and refinement. Neu-tral-atom form factors from International Tables for

[^0]X-ray Crystallography (1968) were used. Corrections for anomalous dispersion were employed for all atoms using the values published by Cromer (1965). Individual weights, $1 / \sigma^{2}\left(F_{o}\right)$, based on counting statistics were used during the course of the refinements. Five reflexions were excluded from the least-squares calculations because they seemed to be affected strongly by the parallel-grown crystal fragment.

The atomic positions, anisotropic temperature factors and the occupancies are listed in Table 2. The observed and the computed structure factors are presented in Table 3.

## Discussion of the structure

The interatomic distances and angles computed by the program ORFFE (Busing, Martin \& Levy, 1964) are given in Table 4. The structure is illustrated in Fig. 2.

The formula of this material is expressed as $\left(\mathrm{Pb}_{1-x}\right.$, $\left.\mathrm{Bi}_{x}\right) \mathrm{Bi}_{2} \mathrm{Cu}_{2} \mathrm{Cu}_{2-x} \mathrm{~S}_{5} \mathrm{I}_{2}$ and that of the crystal used for the structure determination corresponds to $x=0.88$.

Since $(\mathrm{Pb}, \mathrm{Bi})$ is slightly shifted from a mirror plane, it is statistically distributed between two positions with a separation of $0.44 \AA$. Six sulphur atoms around $(\mathrm{Pb}, \mathrm{Bi})$ form a trigonal prism and two more bonds to sulphur atoms are arranged in the directions perpendic-

Table 4. Interatomic distances and angles
Estimated standard deviations are in parentheses.
Superscript

| None | $x$ | $y$ | $z$ |
| :---: | ---: | ---: | ---: |
| i | $-\frac{1}{2}+x$ | $y$ | $\frac{1}{2}-z$ |
| ii | $\frac{1}{2}-x$ | $-y$ | $\frac{1}{2}+z$ |
| iii | $x$ | $y$ | $-z$ |
| iv | $-\frac{1}{2}+x$ | $y$ | $-\frac{1}{2}-z$ |
| v | $x$ | $\frac{1}{2}-y$ | $z$ |
| vi | $-\frac{1}{2}+x$ | $\frac{1}{2}-y$ | $\frac{1}{2}-z$ |
| vii | $x$ | $y$ | $1+z$ |

ular to the prism faces. This type of coordination is very common for lead atoms in many substances. However, the ratio of the occupancies, $m_{\mathrm{Bi}}: m_{\mathrm{Pb}}$, is $0 \cdot 88: 0 \cdot 12$ in the crystal studied and the space around $(\mathrm{Pb}, \mathrm{Bi})$ is probably too large for short Bi-S distances. This misfit of size should cause the statistical distribution of the $(\mathrm{Pb}, \mathrm{Bi})$. The distances between $(\mathrm{Pb}, \mathrm{Bi})$ and the three nearest sulphur atoms, being around $2.67 \AA$, coincide well with those values in Bi sulphosalts (Kupčik, 1972). The coordination of these three sulphur atoms to $(\mathrm{Pb}$, $\mathrm{Bi})$ is trigonal pyramidal.
Bi has three nearest S atoms at distances of 2.56 and $2 \cdot 66 \AA(\times 2)$, and forms with them a flat trigonal pyramid. One copper atom, $\mathrm{Cu}(2)$, and four iodine atoms also surround the bismuth atom at distances of $3 \cdot 20$, $3.42(\times 2)$ and $3.79 \AA(\times 2)$. An idealized drawing of the coordinations around Bi is illustrated in Fig. 3 together with those of bismuth atoms in $\mathrm{Bi}\left(\mathrm{Bi}_{2} \mathrm{~S}_{3}\right)_{9} \mathrm{I}_{3}$ (Miehe \& Kupčík, 1971) and BiSI (Haase-Wessel, 1973).

There are two independent copper atoms in the structure. $\mathrm{Cu}(1)$ is in nearly trigonal planar coordination with S and the mean $\mathrm{Cu}-\mathrm{S}$ distance is $2 \cdot 28 \AA$. This atom is not, however, on the triangle formed by the three sulphur atoms. The displacement of $\mathrm{Cu}(1)$ from the plane is about $0.4 \AA$ and the sum of the three $\mathrm{S}-\mathrm{Cu}-\mathrm{S}$ angles is $350 \cdot 7^{\circ} \cdot \mathrm{Cu}(2)$ is in a tetrahedron, formed by two sulphur and two iodine atoms, with an occupancy 0.56 . The mean $\mathrm{Cu}-\mathrm{S}$ distance is $2 \cdot 26 \AA$. The $\mathrm{Cu}-\mathrm{I}$ distance, $2.66 \AA$, coincides with the published $\mathrm{Cu}-\mathrm{I}$ distance of fourfold coordination ( $2 \cdot 62 \AA$, International Tables for X-ray Crystallography, 1968) Table 5 lists some typical short $\mathrm{Cu}-\mathrm{Cu}$ distances for comparison. The shortest distance between two copper atoms is $2.78 \AA$ in this compound.
The structure of $\left(\mathrm{Pb}_{1-x}, \mathrm{Bi}_{x}\right) \mathrm{Bi}_{2} \mathrm{Cu}_{2} \mathrm{Cu}_{2-x} \mathrm{~S}_{5} \mathrm{I}_{2}$ consists of sheets parallel to (010). These layers are connected to each other by weak $\mathrm{Bi}-\mathrm{I}$ bonds ( $3 \cdot 79 \AA$ ).

Table 4 (cont.)

| Distances ( $\AA$ ) |  |
| :---: | :---: |
| ( $\mathrm{Pb}, \mathrm{Bi}$ ) $-\mathrm{S}(2)$ | 2.655 (9) |
| $(\mathrm{Pb}, \mathrm{Bi})-\mathrm{S}(1)$ | 2.682 (6) |
| $(\mathrm{Pb}, \mathrm{Bi})-\mathrm{S}\left(2^{\prime}\right)$ | 3.083 (9) |
| $(\mathrm{Pb}, \mathrm{Bi})-\mathrm{S}\left(3^{1}\right)$ | $3 \cdot 244$ (6) |
| $(\mathrm{Pb}, \mathrm{Bi})-\mathrm{Cu}\left(1^{\text {i }}\right.$ ) | $3 \cdot 412$ (6) |
| $(\mathrm{Pb}, \mathrm{Bi})-\mathrm{S}\left(3^{\text {vi }}\right)$ | 3.468 (6) |
| $\mathrm{Bi}-\mathrm{S}\left(3^{\text {i }}\right.$ ) | $2 \cdot 563$ (6) |
| $\mathrm{Bi}-\mathrm{S}(2)$ | $2 \cdot 659$ (5) |
| $\mathrm{Bi}-\mathrm{Cu}\left(2^{\text {i }}\right.$ ) | $3 \cdot 204$ (8) |
| $\mathrm{Bi}-\mathrm{I}^{\text {i }}$ | $3 \cdot 417$ (3) |
| $\mathrm{Bi}-\mathrm{I}^{\text {ii }}$ | 3.789 (3) |
| $\mathrm{Cu}(1)-\mathrm{S}(1)$ | 2.267 (8) |
| $\mathrm{Cu}(1)-\mathrm{S}(3)$ | $2 \cdot 285$ (4) |
| $\mathrm{Cu}(1)-\mathrm{Cu}(2)$ | 2.781 (8) |
| $\mathrm{Cu}(1)-\mathrm{Cu}\left(1^{\text {v }}\right.$ ) | $3 \cdot 020$ (11) |
| $\mathrm{Cu}(2)-\mathrm{S}(2)$ | $2 \cdot 140$ (11) |
| $\mathrm{Cu}(2)-\mathrm{S}(3)$ | $2 \cdot 387$ (11) |
| $\mathrm{Cu}(2)-\mathrm{I}$ | $2 \cdot 661$ (7) |


| Angles ( ${ }^{\circ}$ ) |  |
| :---: | :---: |
| $\mathrm{S}(2)-(\mathrm{Pb}, \mathrm{Bi})-\mathrm{S}(1)$ | 84.5 (2) |
| $\mathrm{S}(1)-(\mathrm{Pb}, \mathrm{Bi})-\mathrm{S}\left(1^{\text {vii }}\right)$ | $96 \cdot 8$ (3) |
| $\mathrm{S}(2)-(\mathrm{Pb}, \mathrm{Bi})-\mathrm{S}\left(3^{\text {i }}\right.$ ) | 74.5 (2) |
| $\mathrm{S}\left(3^{1}\right)-(\mathrm{Pb}, \mathrm{Bi})^{\mathbf{i j i}}-\mathrm{S}\left(3^{\text {iv }}\right.$ ) | $76 \cdot 4$ (2) |
| $\mathrm{S}(1)-(\mathrm{Pb}, \mathrm{Bi})-\mathrm{S}\left(3^{\text {i }}\right)$ | $89 \cdot 9$ (2) |
| $\mathrm{S}\left({ }^{\text {i }}\right.$ ) $-\mathrm{Bi}-\mathrm{S}(2)$ | $87 \cdot 1$ (2) |
| $\mathrm{S}(2)-\mathrm{Bi}-\mathrm{S}\left(2^{1 i \mathrm{i}}\right)$ | $97 \cdot 9$ (2) |
| $\mathrm{S}\left(3^{1}\right)-\mathrm{Bi}-\mathrm{I}^{1}$ | $82 \cdot 2$ (1) |
| $\mathrm{I}^{\mathbf{1}}-\mathrm{Bi}-\mathrm{I}^{\text {iv }}$ | 71.9 (1) |
| $\mathrm{S}(2)-\mathrm{Bi}-\mathrm{I}^{\text {i }}$ | 94.2 (1) |
| $\mathrm{S}(1)-\mathrm{Cu}(1)-\mathrm{S}(3)$ | 114.0 (2) |
| $\mathrm{S}(3)-\mathrm{Cu}(1)-\mathrm{S}\left(3^{\text {iif }}\right)$ | 122.7 (3) |
| $\mathrm{S}(2)-\mathrm{Cu}(2)-\mathrm{S}(3)$ | 123.0 (5) |
| $\mathrm{S}(2)-\mathrm{Cu}(2)-\mathrm{I}$ | $112 \cdot 3$ (3) |
| $\mathrm{S}(3)-\mathrm{Cu}(2)-\mathrm{I}$ | 104.1 (2) |
| $\mathrm{I}-\mathrm{Cu}(2)-\mathrm{I}^{\mathrm{ViI}}$ | $97 \cdot 8$ (3) |



Fig. 2. Projection of the structure along the $c$ axis. Thin circles indicate the atoms at $z=0$ and thick circles the atoms at $z=\frac{1}{2}$


Fig. 3. Comparison of the coordinations around Bi in (a) $\mathrm{Bi}\left(\mathrm{Bi}_{2} \mathrm{~S}_{3}\right)_{9} \mathrm{I}_{3}$, (b) BiSI and (c) $\left(\mathrm{Pb}_{1-x}, \mathrm{Bi}_{x}\right) \mathrm{Bi}_{2} \mathrm{Cu}_{2} \mathrm{Cu}_{2-x} \mathrm{~S}_{5} \mathrm{I}_{2}$. Distances from bismuth atoms to the corresponding atoms are indicated.

Table 5. Typical short $\mathrm{Cu}-\mathrm{Cu}$ distances in some compounds

| Cu (metal, at $20^{\circ} \mathrm{C}$ ) | $2.556 \AA$ |
| :---: | :---: |
| $\underset{\text { [diaminobenzene copper(I)] }{ }^{(a)}}{2.45}$ |  |
|  |  |
| $\mathrm{Cu}_{2} \mathrm{~S}$ (chalcocite) | $2.53 * \dagger$ |
| $\mathrm{CuBi}_{5} \mathrm{~S}_{8}{ }^{\text {b }}$ ) | $2 \cdot 56$ |
| $\mathrm{Bi}_{2} \mathrm{Cu}_{3} \mathrm{~S}_{4} \mathrm{Cl}_{1}{ }^{(c)}$ | 2.79* |
| $\mathrm{Cu}_{2}\left(\mathrm{O}_{2} \mathrm{CCH}_{3}\right)_{4} \cdot 2 \mathrm{H}_{2} \mathrm{O}$ | $2 \cdot 614$ |
| [dinuclear copper(II) acetate |  | [dinuclear copper(II) acetate monohydrate] ${ }^{(d)}$

References: (a) Brown \& Dunitz (1961). (b) Ohmasa \& Nowacki (1973). (c) Lewis \& Kupčík (1974). (d) Brown \& Chidambaram (1973).

* The shortest distances are listed.
$\dagger$ This value was calculated from the data of Evans (1971).

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[^0]:    * A version to calculate the effect of anomalous dispersion.

